Alpha dose to a thin layer

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With increasing success in ESR dating of shells and teeth it becomes topical to consider the alpha dose within thin layers. This is done here using the approximation of constant effect (TL or ESR) per unit length of track, which is the basis of the three current systems of assessing alpha dose: *a*-value (Aitken and Bowman, 1975), *b*-value (Bowman and Huntley, 1984) and omnidirectional flux (Guérin and Valladas, 1980). The dose in terms of energy deposited increases as the particle slows down but because of a compensatory decrease in TL effectiveness it turns out that constant TL per unit length of track is a good approximation; it has now been shown that the same is true for ESR (see accompanying note by Lyons).

Dose in inert medium

First we consider the effective dose delivered by a radioactive medium to an inert medium, the media being separated by a plane boundary of infinite extent; see Fig. 1. Let the alpha particles be emitted with initial range R_o, the same in both mediums. Then the residual range carried by a particle across a plane at depth x in the inert medium is $(R_o - R)$ where,

$$R = (h+x) / \cos\theta$$

and h is the distance of the point of emission from the boundary, with θ being the angle to the vertical.

If N is the number of particles emitted per unit volume then the total residual range carried across the plane by particles emitted from unit volume at distance h from the boundary with paths lying between θ and $\theta + d\theta$ is given by

$$\frac{2\pi r\ dr \cos\theta\ (R_o - R)\ N}{4\pi R^2} \qquad \text{...... (1)}$$

which may be written as

$$\frac{N\ (h+x)(R_o - R)\,dR}{2R^2} \qquad \text{...... (2)}$$

since $\cos\theta = \frac{(h+x)}{R}$ and $r\ dr = R\ dR$

The total residual range carried across the plane by particles emitted from the unit volume is obtained by integration, between the limits of $R = (h+x)$ and $R = R_o$, as,

$$\frac{N}{2}\{R_o - (h+x) - (h+x)\ln R_o + (h+x)\ln(h+x)\} \qquad \text{...... (3)}$$

as long as $(h+x) \le R_o$.

We now integrate over the radioactive medium and obtain the residual range delivered across the plane by particles emitted from a cylinder of area A and height $(R_o - x)$ as

$$\frac{NA}{8}(R_o^2 - 4xR_o + 3x^2 + 2x^2\ln R_o - 2x^2\ln x) \qquad \text{...... (4)}$$

with the restrictions that $x \le R_o$, and $A \gg R_o^2$. From this expression we may derive the effective alpha dose in the inert medium for various situations.

Case A: Inert medium of thickness $t > R_o$.

Substituting $x = 0$ into (4), the tracklength entering the inert medium is obtained as $NAR_o^2/8$ and hence the average effective dose, ie the track length per unit volume, is

$$\frac{MR_o}{8t} \qquad \text{...... (5)}$$

where $M = NR_o$ is the effective dose within the radioactive medium for $h > R_o$, ie the infinite matrix dose.

Case B: Inert medium of thickness $t \le R_o$

The track length that exits across the plane at $x = t$ is obtained by substitution into (4), whence the average effective dose between $x = 0$ and $x = t$ is

$$\frac{M}{2}\{1 - \frac{3t}{4R_o} - \frac{t}{2R_o}\ln(\frac{R_o}{t})\} \qquad \text{...... (6)}$$

Case C: Inert medium of thickness t from which a surface layer I is removed before measurement.

By subtracting the track length exiting across $x = t$ from that entering across $x = I$ the average dose is obtained as

$$\frac{M}{2}\{1 - \frac{3(t + I)}{4R_o} - \frac{(t + I)\ln R_o}{2R_o} + \frac{(t^2 \ln t - I^2 \ln I)}{2R_o(t - I)}\}$$

...... (7)

with the restriction that $t \leq R_o$. If $t \geq R_o$ then there is no exiting of track length and the average dose is

$$\frac{M}{2}\{\frac{R_o}{4(t - I)} - \frac{I}{(t - I)} + [\frac{I^2}{4R_o(t - I)}(3 + 2\ln(\frac{R_o}{I}))]\}$$

...... (8)

The effective dose at depth x, obtained through differentiation of (4) is

$$\frac{M}{2}\{1 - \frac{x}{R_o} - \frac{x}{R_o}\ln(\frac{R_o}{x})\} \qquad \text{...... (9)}$$

This is illustrated in Fig. 2 as curve (a); curve (b) shows the average dose, illustrating Cases A and B. In table 1 numerical values are given for all three cases.

In application the expressions should be summed over the individual alpha particle emissions of the thorium and uranium chains (see accompanying note by Grün). An approximate result can be obtained by using the average value of R_o for each chain, *viz* 0.026 mm for the thorium series and 0.021 mm for the uranium series, both in equilibrium; these values assume, for both media, a density of 2.7 g/cm^3 and a stopping power equal to that of pottery as given by Bowman (1982) who also indicates that the stopping power of calcium carbonate is close to that of pottery. In practical application the values of R_o are liable to be different between the two media on account of differing densities; the best approximation is then given by using the value corresponding to the inert medium.

Dose in radioactive medium

The residual range exiting from each surface of a layer of thickness H ($\leq R_o$) of radioactive medium sandwiched between inert media is obtained by putting $x = 0$ in expression (3) and integrating from $h = 0$ to $h = H$, ie

$$\frac{NR_oH}{2}\{1 - \frac{3H}{4R_o} - \frac{H}{2R_o}\ln(\frac{R_o}{H})\} \qquad \text{...... (10)}$$

By doubling this and subtracting from the residual range generated in the layer the average effective dose is obtained as

$$M\{\frac{3H}{4R_o} + \frac{H}{2R_o}\ln(\frac{R_o}{H})\}$$

..... (11)

For the case when $H > R_o$ the residual range exiting from each surface is obtained by substituting R_o for H in (10), ie ($NR_o^2/8$). Hence the average effective dose is

$$M\{1 - \frac{R_o}{4H}\} \qquad \text{...... (12)}$$

Because, for alpha particles, backscattering effects are negligible the dose in the radioactive medium is not influenced by the stopping power or density of the inert medium and the assumption that R_o is the same in both media.

Table 1.
Effective alpha doses (expressed as fraction of infinite matrix dose)

(x/R_o)	Average dose for $t = x$	Dose at depth x	Percentage removed
0.01	0.485	0.472	3.9
0.02	0.473	0.451	7.6
0.03	0.463	0.433	11
0.04	0.453	0.416	15
0.05	0.444	0.400	18
0.1	0.405	0.335	32
0.15	0.373	0.283	45
0.2	0.345	0.239	55
0.25	0.320	0.202	64
0.3	0.297	0.170	71
0.4	0.259	0.117	83
0.5	0.226	0.077	90
0.6	0.199	0.047	95
0.7	0.175	0.025	98
0.8	0.156	0.011	99
0.9	0.139	0.003	
1.0	0.125	0	
1.5	0.083		
2	0.063		
2.5	0.050		
3	0.042		
4	0.031		
5	0.025		

Note: 'Percentage removed' gives the percentage decrease in the average dose in a layer of thickness $t = R_o$ due to removal of $I = x$.

Acknowledgement
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References

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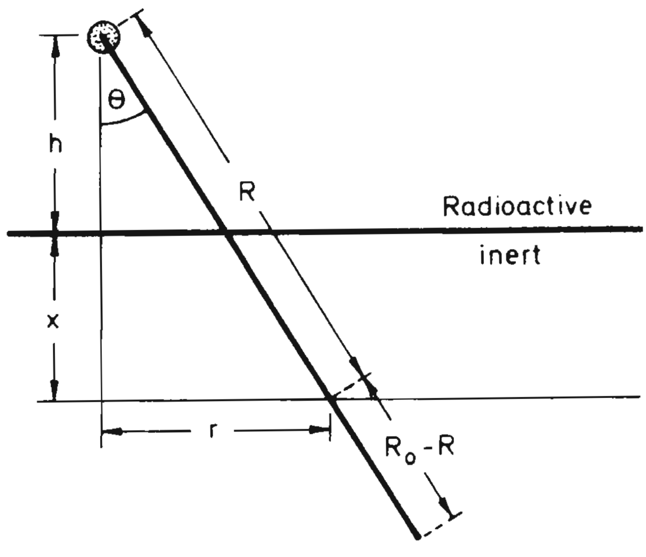

Figure 1
Alpha track crossing boundary between radioactive medium and inert medium; the initial range R_o is assumed to be the same in both media. The residual range crossing an imaginary plane at depth x is R_o- R.

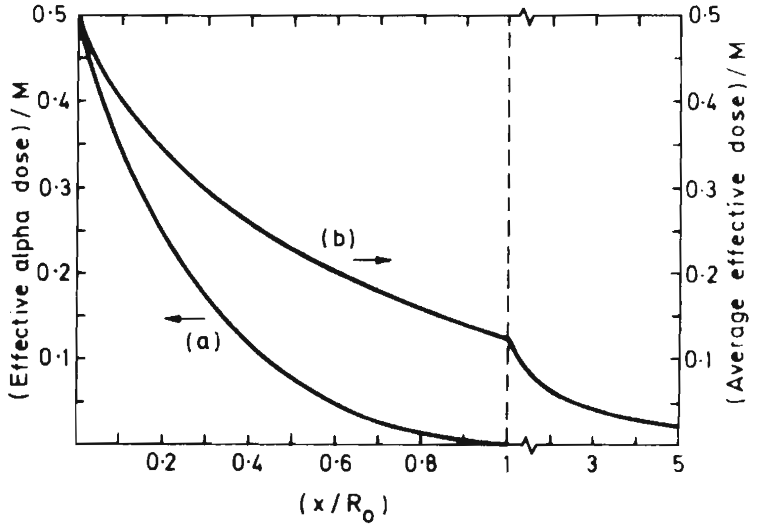

Figure.2
Effective alpha dose in surface layer of inert medium close to its plane boundary with radioactive medium in which the infinite matrix effective alpha dose is M; the distance into the inert medium is x and the initial range is R_O. Note change of horizontal scale at $(x/R_O) = 1$. Curve (a) represents the dose at distance x ; curve (b) represents the average dose between x = 0 and x .