Pairs precision required in alpha counting

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Introduction

In thick source alpha counting of samples and soil for TL dating an estimate of the counts due to the thorium chain can be determined from the number of coincident 'pairs' that are observed. A coincidence window of 0.38s is commonly used and after subtraction of random coincidences the remaining 'true pairs' are due to the rapid decay of Po-216, halflife 0.145s, following its formation from Rn-220, both being alpha emitters. For samples in which the thorium activity is equal to the uranium activity the ratio of total counts to true pairs is about 40:1. Hence to obtain 100 pairs (and hence get ±10% statistical uncertainty) a counting time of about a week is necessary for a sample having a count-rate (for a 42-mm detector) of 10 per ks and proportionally longer for samples of weaker radioactivity.

The purpose of this note is to point out that acceptably precise values for annual beta and gamma doses can be obtained with substantially fewer pairs than 100. Perhaps many practitioners realize this already but to others it may be useful. Of course for alpha dose there is only slight dependence on thorium/uranium ratio.

Beta dose

Using data from Nambi and Aitken (1986) the annual beta dose, in µGy, is given by

$$\begin{aligned} D_\beta &= 86.6\,\alpha_u + 57.3\,\alpha_h \\ &= 86.6\,\alpha - 29.3\,\alpha_h \end{aligned} \qquad (1)$$

where α_u and α_h are the counts/ks from the U and Th chains respectively, using a 42-mm detector with the usual electronic threshold setting of 85% for a thorium-only sample and $\alpha = (\alpha_u + \alpha_h)$.

In the case of a coincidence window that is open from 0.02 to 0.04 s following each count (as in Daybreak and in Littlemore equipment) the count-rate due to the thorium chain is related to the count-rate of true pairs, p, according to

$$\alpha_h = 21.1p \qquad (2)$$

and the true pairs are obtained from the observed doubles rate, d, by subtraction of the random coincidence rate, r, accordingto

$$p = d - r \qquad (3)$$

Substituting from (2) into (1),

$$D_\beta = 86.6\,\alpha - 618\,p \qquad (4)$$

The uncertainty in the first term is $86.6(\alpha/t)$ and in the second, $618\,[(d + r)/t]^{0.5}$ where $\alpha = \alpha_u + \alpha_h$, and t ks is the duration of the count. Assuming these uncertainties to be uncorrelated, the uncertainty in D_β is given by

$$(\delta D_\beta)^2 = 86.62^2/t + 618^2\,(d + r)/t \qquad (5)$$

For a coincidence time of 0.38s, $r = \alpha^2/2631$, and writing $C = r/\alpha$ and $B = d/\alpha$, the fractional uncertainty is

$$\left(\frac{\delta D_\beta}{D_\beta}\right)^2 = \frac{1}{\alpha t}\cdot\frac{86.6^2 + 618^2\,(B + C)}{\{86.6 - 618\,(B - C)\}^2} \qquad (6)$$

As an illustrative example, take $\alpha = 10\ \mathrm{ks}^{-1}$, in which case C = 0.004; suppose for this sample B = 0.03, then to obtain ± 5% uncertainty in D_β, eqn. (6) indicates requirement for a total count (αt) of 1650 and a doubles count of 50. This contrastswith the commonly held assumption that an uncertainty even as poor as ±10% requires 100 doubles. Inspection of eqn. (6) shows that there is comparative insensitivity to the value of C in the denominator and for convenience in judging whether sufficient doubles have been obtained during a count, (B - C) in the denominator may be taken as equal to (B + C); this has been done in constructing fig. 1a.

Gamma dose

In a similar manner, starting from the expression (from Nambi and Aitken) for the annual gamma dose as

$$\begin{aligned} D_\beta &= 67.0\,\alpha_u + 104.7\,\alpha_h \\ &= 67.0\,\alpha_u + 37.7\,\alpha_h \end{aligned} \qquad (7)$$

the fractional uncertainty may be written as

$$\left(\frac{\delta D_\gamma}{D_\gamma}\right)^2 = \frac{1}{\alpha t}\cdot\frac{67^2 + 796^2\,(B + C)}{\{67 - 796\,(B - C)\}^2} \qquad (8)$$

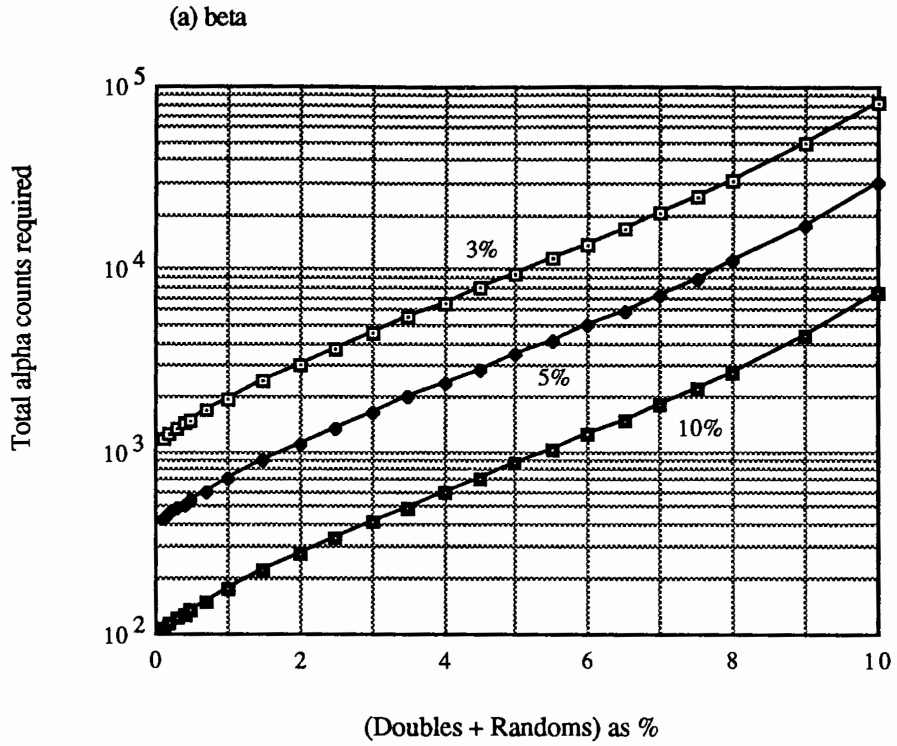
This is illustrated in fig.1b, again with approximation for (B - C) in the denominator.

Combined beta and gamma dose: alpha dose

From equations (1) and (7) we have

$$\begin{aligned} D_\beta + D_\gamma &= 153.6\,\alpha_u + 162.0\,\alpha_h \\ &= 153.6\,\alpha_u + 8.4\,\alpha_h \end{aligned} \qquad (9)$$

From this we see that the extreme assumptions of $\alpha_h = 0$ and $\alpha_h = \alpha$ both give combined doses which are within 3% of the middle value obtained by putting $\alpha_h = 0.5\alpha$. Hence there is no need for pairs counting in cases where the combined dose is relevant (see Discussion).



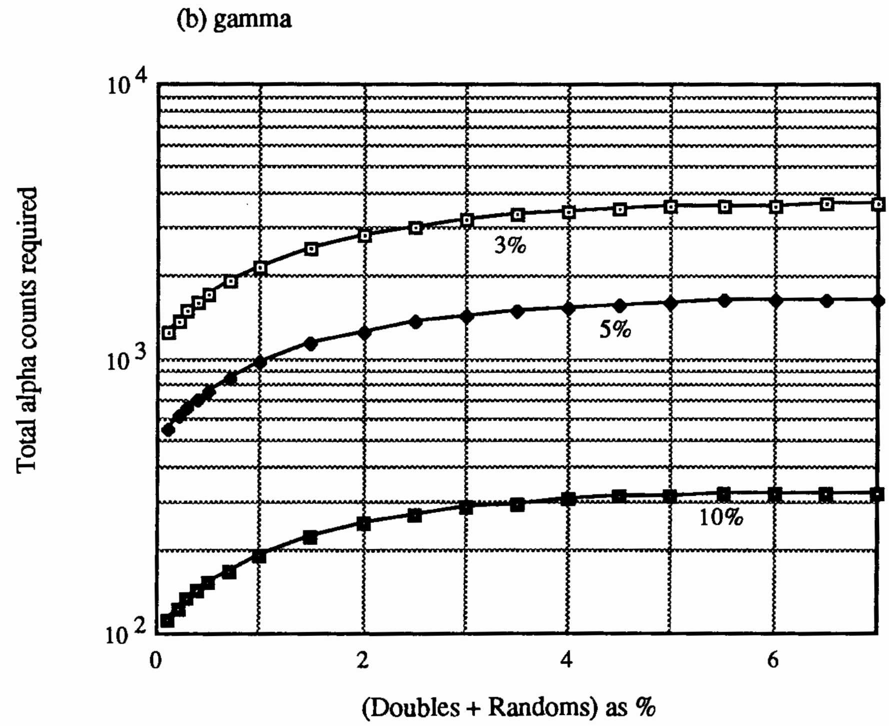


Figure 1.
Total counts (singles plus doubles) required to give precision indicated. The horizontal scale represents (doubles+ randoms) expressed as a percentage of total counts. Coincidence time: 0.38s. (a) For annual beta dose; (b) For annual gamma dose.

The annual alpha dose also is only weakly dependent on the Th/U ratio, being given by

$$D_\alpha = 1645\,\alpha_u + 1480\,\alpha_h \qquad (10)$$

$$= 1645\,\alpha_u + 166\,\alpha_h \qquad (11)$$

In this case the doses corresponding to the extreme assumptions lie within 5% of the dose corresponding to $\alpha_h = 0.5\,\alpha$.

Thorium/uranium ratio

This ratio, in terms of ppm, is given by

$$R = \frac{1.70}{0.499} \cdot \frac{\alpha_h}{\alpha - \alpha_h} \qquad (12)$$

the numerical factors, as elsewhere, being taken from Nambi and Aitken (1986). On the assumption, as before, that statistical fluctuations in α and α_h are uncorrelated, the fractional uncertainty in R is given by

$$\left(\frac{\delta R}{R}\right)^2 = \left(\frac{\alpha}{\alpha - 21.1p}\right)^2 \cdot \left(\frac{(\delta\alpha)^2}{\alpha^2} + \frac{(\delta p_h)^2}{p^2}\right) \qquad (13)$$

where

$$\frac{(\delta\alpha)^2}{\alpha^2} = \frac{1}{\alpha t}$$

and

$$\frac{\delta p^2}{p^2} = \frac{1}{\alpha t} \cdot \frac{(B + C)}{(B - C)^2}$$

For the numerical values following equation (6) the second term on the right-hand side is equal to 0.025^2, the first to 0.175^2, and the fractional error in R is 0.39. This is in strong contrast to the ±5% uncertainty in beta dose that is obtained. In order to obtain even ±10% uncertainty in R with this sample a doubles count of 645 is required, needing a counting time of a month.

Discussion

The foregoing considers only the uncertainties arising from counting statistics, the systematic uncertainties associated with the possibility of radon escape being ignored. However the purpose of the analysis is to indicate that excessively long counting times are unnecessary unless the ratio Th/U is required.

The considerations regarding gamma dose are relevant only in circumstances where there is radioactive homogeneity in a sphere of radius 30 cm around the sample. The considerations regarding combined beta-plus-gamma dose are relevant only in the case of sediment dating in such circumstances. In all other circumstances the gamma dose needs to be determined by on-site measurements.

Screen efficiency

It is often assumed that there is no batch-to-batch variability in composite ZnS screens. While this is usually the case we have recently had a batch of screens in which the counting efficiency was low by 25% compared to normal. Hence routine checking of screen efficiency is important; this can be done conveniently by means of a source made up of pitchblende in resin (eg "Plasticraft"). This should be of the same diameter as the screen and hence give a good replication of actual sample geometry. With about 5 g of pitchblende in 5 g of resin the count-rate is about 300 per sec, enabling good statistical precision to be obtained in a 100-second count. Use of a thin source (eg Am-241 or Ra-228) is misleading because the spectrum differs grossly from the thick source spectrum emitted by a sample.

Erratum

I would like to take this opportunity of apologizing for an error on p. 87 of Aitken (1985), kindly pointed out to me by Stephen Stokes. The upper limit for no-sample screen-only count-rate should have been given as 0.1 per ks not 0.01.

References

Aitken, M J (1985) *Thermoluminescence dating*. Academic Press, London, Florida.

Nambi, K S V, Aitken, M J (1986)Annual dose conversion factors for TL and ESR dating, *Archaeometry*, **28**, 202-205.

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