

Absorbed dose fraction for ^{87}Rb β particles

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Abstract: The dose absorbed by spherical quartz grains containing rubidium is calculated for grain diameters from 5 μm to 100 μm . The equivalent geometric factor as a function of grain diameter is also given.

Introduction

Recently it was stated (Huntley and Hancock, 2001) that a clear need exists for a proper calculation of the absorbed energy fraction for β particles emitted by ^{87}Rb from within grains of various sizes. The tools required for such a calculation have previously been published (Readhead, 1987; with an *erratum* noted in *Ancient TL* 6, p.20), but as this work was not referenced in the review by Adamiec and Aitken (1998), readers may not be familiar with it. A synopsis of the method is provided below, along with the results.

Theory

Spencer (1959) calculated the average energy dissipated near point isotropic sources of mono-energetic electrons and presented his numerical results in the form of a “de-dimensionalized energy distribution” $J(x)$, where x is the distance from the source in units of pathlength r_0 . He calculated electron penetration taking into account both nuclear elastic scattering and electron slowing down, but did not include straggling. Charlton (1970) states that Spencer’s results have been extensively tested by Cross (1967), and that “over a wide range of beta spectra there is excellent agreement between theory and experiment, except at distances from the source greater than about half of the range of the most energetic electrons in the spectrum. Within this distance about 95% of the total source energy is deposited.”

The calculations of Spencer are the basis of absorbed dose distribution functions derived by both Berger (1971, 1973) and Charlton (1970). The method of Berger has previously been used by Mejdahl (1979); that of Charlton by Bell (1978, 1979). Both are equivalent and Bell (1979) found good agreement between his and Mejdahl’s results. The approach of Charlton is followed here.

Consider a sphere of diameter D containing a homogeneous distribution of point sources, surrounded by a region not containing any sources, as in Figure 1.

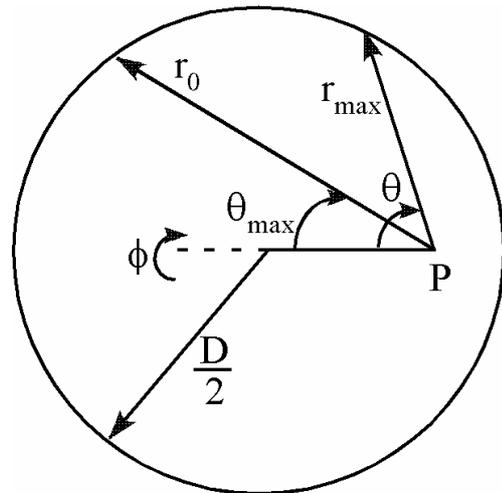


Figure 1.

Geometry for a spherical absorber which emits N_0 electrons per unit mass. It is surrounded by a region which does not emit electrons.

Within the sphere N_0 electrons are emitted per unit mass, each of initial energy E_0 and pathlength r_0 . Using Spencer’s results, Charlton showed that the energy dissipated per unit volume at point P is

$$D_e(r) = N_0 E_0 S_e(r) \quad (1)$$

where (after some manipulation) the spherical geometrical function

$$S_e(r) = 1 - \frac{1}{2} \int_{\theta_{\max}}^{\pi} \left(1 - \frac{\int_0^{r_{\max}} J(x) dx}{\int_0^1 J(x) dx} \right) \sin \theta \, d\theta \quad (2)$$

is obtained. Here D is the diameter of the grain, r is the distance from the centre of the grain to P,

$$r_{\max} = r \cos \theta + \sqrt{\left(\frac{D}{2}\right)^2 - (r \sin \theta)^2} \quad (3)$$

is the distance from P to the edge of the grain for an angle θ and

$$\theta_{\max} = \begin{cases} 0 & \text{for } r_0 > D/2 + r \\ \arccos \left[\frac{r^2 + r_0^2 - (D/2)^2}{2rr_0} \right] & \text{otherwise.} \end{cases} \quad (4)$$

Equation (1) gives the dose absorbed at a point. The mean dose absorbed by a spherical grain is

$$\langle D_e \rangle = N_0 E_0 \langle S_e \rangle \quad (5)$$

where

$$\langle S_e \rangle = \frac{\int_0^{2\pi} \int_0^\pi \int_0^{D/2} S_e r^2 \sin \theta dr d\theta d\phi}{\int_0^{2\pi} \int_0^\pi \int_0^{D/2} r^2 \sin \theta dr d\theta d\phi} \quad (6)$$

When a nuclide decays and emits a β particle, that particle can have a range of possible energies up to a maximum value E_{\max} . The mean dose absorbed from a β particle must be averaged over this spectrum, so

$$\langle D_\beta \rangle = N_0 \langle E_\beta \rangle \quad (7)$$

where

$$\langle E_\beta \rangle = \frac{\int_0^{E_{\max}} \langle S_e \rangle E n(E) dE}{\int_0^{E_{\max}} n(E) dE} \quad (8)$$

$n(E)dE$ is the number of β particles emitted per unit energy with initial energies in the interval E to $E+dE$. For many transitions the energy distribution of the spectrum is not fully known, but Murthy (1971) gives formulae for calculating it.

Computational considerations

To evaluate the above equations the β particle maximum energies and intensities, and internal conversion and Auger electron energies and intensities can be obtained from the Nuclear Data Retrieval Program (<http://www.nndc.bnl.gov/>). The pathlengths in quartz can be derived from the tables of Pages *et al.* (1972).

Spencer's $J(x)$ was evaluated for electrons with initial energies of 0.025, 0.05, 0.1, 0.2, 0.4, 0.7, 1, 2, 4 and 10 MeV traversing selected media, including carbon ($Z=6$) and aluminium ($Z=13$) using a mesh in x of $\Delta x = 0.025$. To interpolate in x requires the use of a quadratic interpolation formulae. To interpolate for quartz at a particular initial energy one uses

$$\frac{\ln J_q(x)}{A_q} = \left[\frac{\ln J_a(x)}{A_a} - \frac{\ln J_c(x)}{A_c} \right] \left(\frac{Z_q - Z_c}{Z_a - Z_c} \right) + \frac{\ln J_c(x)}{A_c} \quad (9)$$

and

$$A_q = A_c + (A_a - A_c) \left(\frac{Z_q - Z_c}{Z_a - Z_c} \right) \quad (10)$$

where the subscripts q, c and a refer to Z, $J(x)$ and constants A (given by Spencer) for quartz, carbon and aluminium, respectively. To interpolate in initial energy one uses:

$$\frac{\ln J(x)}{A} = \left[\frac{\ln J_{UE}(x)}{A_{UE}} - \frac{\ln J_{LE}(x)}{A_{LE}} \right] \left(\frac{\log E - \log E_L}{\log E_U - \log E_L} \right) + \frac{\ln J_{LE}(x)}{A_{LE}} \quad (11)$$

and

$$A = A_{LE} + (A_{UE} - A_{LE}) \left(\frac{\log E - \log E_L}{\log E_U - \log E_L} \right) \quad (12)$$

where E is the energy of interest, bracketed above by E_U and below by E_L , with associated $J_{UE}(x)$ and $J_{LE}(x)$, and A_{UE} and A_{LE} .

For $Z=37$ Murthy gives the β energy spectrum as:

$$n(E)dE = k(1.7964 + 8.4692E + 6.8327E^2)(E_{\max} - E)^2 dE \quad (13)$$

where k is a constant.

Results

For ^{87}Rb the maximum β energy is 0.2823 MeV. Table 1 shows the absorbed dose in MeV/ N_0 and $\mu\text{Gy/a}/(\text{ppm Rb})$ for grain diameters from $5\mu\text{m}$ to 100 mm. Since the values are obtained after averaging over the β spectrum, a simple geometric factor cannot be given. However, an equivalent value is obtained if $\langle S_e \rangle$ is set to 1 in Equation (8), in which case the absorbed dose is 0.0825 MeV/ N_0 or 0.3580 $\mu\text{Gy/a}/(\text{ppm Rb})$. This leads to the equivalent geometric factors shown in the table for each diameter. Of course, for very large grains this same maximum absorbed dose is achieved, as the geometric factor approaches 1. Figure 2 plots the equivalent geometric factor as a function of grain diameter.

Note that the results apply to an absorber of quartz, for which $Z_q \approx 10.8$. Slightly different results will be obtained with other absorbers, by using the appropriate effective atomic number in Equations 9 and 10.

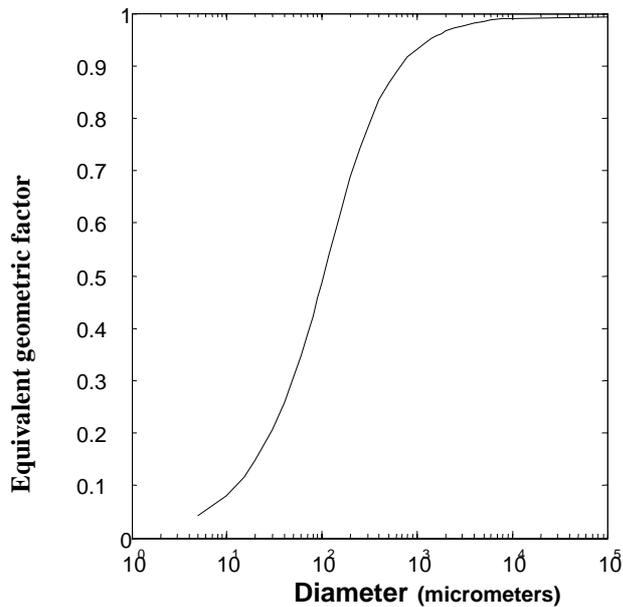


Figure 2.

Equivalent geometric factor as a function of grain diameter for a quartz grain containing an homogeneous distribution of ^{87}Rb

In addition it should be pointed out that the ^{87}Rb β energy spectra is most likely an approximation on the part of Murthy, due to the paucity of experimental data. His paper includes correction factors for various types of forbiddingness, but doesn't specifically mention ^{87}Rb . However, an approximate result based on his spectra is better than no result at all, and is certainly an improvement over that of Bell (1979), who only used the average β energy for ^{87}Rb when calculating the attenuation factor for 100 μm quartz grains.

References

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Diameter (mm)	Absorbed dose (MeV/ N_0)	Absorbed dose $\mu\text{Gy/a}/(\text{ppm Rb})$	Equivalent geometric factor
0.005	0.0036	0.015	0.043
0.01	0.0067	0.029	0.081
0.015	0.0095	0.041	0.115
0.02	0.0121	0.053	0.147
0.03	0.0169	0.073	0.205
0.04	0.0212	0.092	0.257
0.05	0.0251	0.109	0.305
0.06	0.0287	0.125	0.348
0.07	0.0320	0.139	0.388
0.08	0.0350	0.152	0.424
0.09	0.0377	0.164	0.457
0.1	0.0402	0.175	0.488
0.12	0.0447	0.194	0.542
0.14	0.0485	0.211	0.588
0.16	0.0517	0.225	0.627
0.18	0.0545	0.236	0.660
0.2	0.0568	0.247	0.689
0.25	0.0614	0.267	0.744
0.3	0.0646	0.281	0.784
0.4	0.0689	0.299	0.835
0.5	0.0715	0.311	0.867
0.6	0.0733	0.318	0.889
0.8	0.0755	0.328	0.916
1	0.0769	0.334	0.932
1.2	0.0778	0.338	0.943
1.4	0.0785	0.341	0.951
1.6	0.0789	0.343	0.957
1.8	0.0793	0.344	0.962
2	0.0796	0.346	0.965
2.5	0.0802	0.348	0.972
3	0.0805	0.350	0.976
4	0.0809	0.352	0.981
5	0.0812	0.353	0.984
6	0.0814	0.353	0.987
8	0.0816	0.354	0.989
10	0.0817	0.355	0.990
100	0.0819	0.355	0.992

Table 1. Absorbed dose and equivalent geometric factor as a function of grain diameter for a quartz grain containing an homogeneous distribution of ^{87}Rb .

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Reviewer

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